

## COMPARISON AND FUSION OF ODOMETRY AND GPS WITH LINEAR FILTERING FOR OUTDOOR ROBOT NAVIGATION

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**Abstract:** The present paper deals with the comparison and fusion of odometry and absolute Global Positioning System (GPS) with complementary linear filtering for the navigation of an outdoor robot. This system was implemented on a home made mobile robot named Rover Autonomous Navigation Tool (R-ANT). Experimental results are presented, which allow to compare the two original measurements as well as the filtered output, clearly showing the advantages and disadvantages of each of the three localization systems. *Copyright ©Robótica'2003*

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### 1. INTRODUCTION

Location is basic to navigation. So that a mobile robot may autonomously navigate, it needs to know its exact position and orientation. Robot localization is therefore a key issue in providing autonomous capabilities to a mobile robot. The different methods developed to solve this problem can be included in one of two major categories: relative and absolute positioning.

Relative positioning is usually based on odometry, a simple, inexpensive and easy to implement real-time method, where the vehicle position is determined by geometric interpolation according to the angular displacement of the vehicle wheels. This method provides only incremental positioning and is sensitive to error sources that fit into one of the two categories (Borenstein and Feng 1996), systematic (unequal wheel diameters, misalignment of wheels, etc.) and nonsystematic errors (wheel slippage, travel over uneven floors, etc.), having therefore limited accuracy especially as the distance travelled increases. (Borenstein and Feng 1996) and (Goel *et al.* 1999)

present calibration procedures to compensate some of the systematic errors, namely, unequal wheel diameters and uncertainty about the effective wheel base.

Absolute positioning allows a mobile robot to determine its location and velocity independently of previous measurements, i.e., the navigation is made with respect to a coordinate frame based on the environment. In this case, the accumulation of errors does not occur, but the presence of landmarks is needed. The Global Positioning System (GPS) can consistently provide accurate position, velocity and timing information in good satellite signal tracking environments. The main factor limiting the use of GPS is the requirement for line-of-sight between the receiver antenna and the satellites, which cannot always be met.

Absolute and relative localization are complementary. Generally, absolute systems do not provide localization data at a high frequency, whereas relative systems can. Moreover, if an absolute system does not detect enough landmarks for its localization process, relative techniques can calculate estimates during limited periods of time. Combinations of the two approaches can yield very accurate positioning systems. The most common formalism used to associate these two kinds of systems is the Kalman filter

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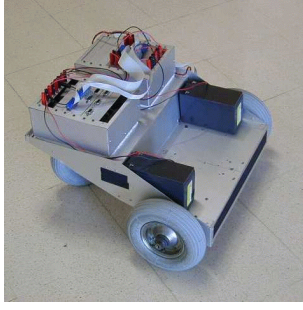


Fig. 1. The R-ANT platform.

(Moutinho and Azinheira 2003), where the observations relating to the motion of the robot, as well as a dynamic model derived from some physical laws are required. However, this model is almost never available at a level to satisfy the basic reliability requirements. In this way, a simple solution was tested and compared with odometry and GPS alone: complementary linear filtering of the odometry and GPS measurements and where no physical modelling is called for. In (Baerveldt and Klang 1997) the same was done for attitude estimation for an autonomous helicopter.

The present paper deals with the comparison and fusion of odometry and absolute GPS with complementary linear filtering for the navigation of a small outdoor robot, where in (Moutinho and Azinheira 2003) fusion results were presented using an extended Kalman filter.

In section 2 the robot model is presented. Section 3 describes the design of the complementary linear filter. The experimental setup and the results obtained are presented in section 4 and section 5 draws some concluding remarks.

## 2. ROBOT ODOMETRY MODEL

The R-ANT platform used for the experiments has two opposed drive wheels, each coupled with a dc motor and an encoder, and one fly wheel for equilibrium, as represented in figure 1. The R-ANT platform has two levels: the lower at the front for the heavier components (motors and batteries) and the higher at the rear for the microprocessor and acquisition/control boards. This way, supporting less weight allows the fly wheel to move more freely, having therefore less interference with the vehicle movement.

This mobile robot has been mostly used for indoor navigation studies but in the present paper it is used as a test platform for outdoor navigation. The very low speed of this robot (with a maximum of 10 cm/s) is adequate to reduce slippage and yield a better odometry estimate but is quite difficult to be tracked by a standard GPS receiver, which would be expecting a higher dynamics or, on the other hand, a static position.

### 2.1 Modeling Odometry

The two front wheels are equipped with encoders. At each sampling period,  $\Delta T$ , the signed integration of the encoder pulses provides an estimate of angular displacement, corresponding to the  $\Delta d^l$  and  $\Delta d^r$  distances travelled during that period by the left and right wheels respectively. The wheels speed measured for each instant  $k$  is given by

$$V_k^{l,r} = \frac{\Delta d_k^{l,r}}{\Delta T} \quad (1)$$

The corresponding translation ( $\Delta s$ ) and rotation ( $\Delta \theta$ ), measured with respect to the mid-point of the axle, are given by

$$\Delta s_k = \Delta T \frac{\Delta V_k^l + \Delta V_k^r}{2} \quad (2)$$

$$\Delta \theta_k = \Delta T \frac{\Delta V_k^r - \Delta V_k^l}{b} \quad (3)$$

where  $b$  is the distance between the two driving wheels.

In a two-dimensional space and assuming linear displacement at each sampling time, the location of the R-ANT at step  $k$  can be represented by

$$x_k \simeq x_{k-1} + \Delta s_k \cos(\theta_{k-1} + \frac{\Delta \theta_k}{2}) \quad (4)$$

$$y_k \simeq y_{k-1} + \Delta s_k \sin(\theta_{k-1} + \frac{\Delta \theta_k}{2}) \quad (5)$$

$$\theta_k \simeq \theta_{k-1} + \Delta \theta_k \quad (6)$$

where  $x$  and  $y$  denote the position of the center of the axle in a Cartesian ground frame, and  $\theta$  is the angle between the vehicle longitudinal axis and the  $x$  axis. For the experiments reported here, the reference ground frame is chosen in such a way that the robot starts at the origin and is initially aligned with the  $y$  direction.

According to (Wang 1988), with the added assumption of a circular path, the second part of equations (4) and (5) should be multiplied by the adjustment factor in equation (7).

$$f_{aj} = \frac{\sin(\Delta \theta_k/2)}{\Delta \theta_k/2} \quad (7)$$

In our case this factor does not bring any change because the sampling time (0.25s) is too small when compared with the angular speed of the vehicle.

## 3. FILTER DESIGN

Position, orientation and velocity measurements of the mobile robot R-ANT were obtained by odometry and GPS. The goalpoint now is to combine all the information from these redundant measurements of the same

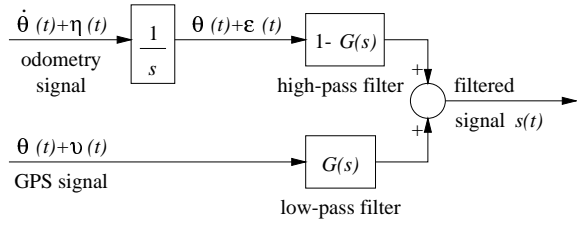


Fig. 2. Conceptual complementary filter for combining odometry and GPS signals.

signals in such a way as to minimize the instrumentation errors. A method of filtering the noise without distorting the signal is complementary filter (Brown and Hwang 1997). For instance, consider the robot has angular velocity  $\dot{\theta}$  and one wants to know its exact orientation  $\theta$ . GPS, an absolute positioning system, provides at each moment orientation values with high-frequency noise, while odometry, a dead-reckoning positioning system, provides noisy orientation values by integrating at each moment noisy angular velocity. From the block diagram in figure 2, the Laplace transform of the output  $s(t)$  may be written as

$$S(s) = \theta(s) + \varepsilon(s)[1 - G(s)] + v(s)G(s) \quad (8)$$

where  $v$  and  $\varepsilon$  are respectively noise inputs on GPS position and odometry velocity measurements.

Clearly, the signal term  $S(s)$  is not affected by the choice of  $G(s)$  in any way. On the other hand, the two noise inputs are modified by the complementary transfer functions  $[1 - G(s)]$  and  $G(s)$ . Because  $v$  is predominantly high-frequency noise and  $\varepsilon$  low-frequency, choosing  $G(s)$  to be a low-pass filter will automatically attenuate  $v$  as well as  $\varepsilon$ . A first order low-pass filter is of the form

$$G(s) = \frac{1}{\Gamma s + 1} \quad (9)$$

It is now necessary to adjust the time constant  $\Gamma$  to minimize the effects of the noise sources  $v$  and  $\varepsilon$ , as well as define the order of the filter.

An approximate model for the measurement system with the two position sensors will now be made. The relation between the angular velocity  $\dot{\theta}$  of the robot and the voltage  $U$  sent to the motors is given by

$$\frac{\dot{\theta}(j\omega)}{U(j\omega)} = \frac{k}{1 + Tj\omega} \quad (10)$$

where  $k$  and  $T$  are the gain and time constant of the system respectively. The orientation is obtained by integration of the angular velocity:

$$\frac{\theta(j\omega)}{U(j\omega)} = \frac{1}{j\omega} \frac{k}{1 + Tj\omega} \quad (11)$$

The measurements  $\theta_o$  and  $\theta_g$  provided by odometry and GPS respectively are

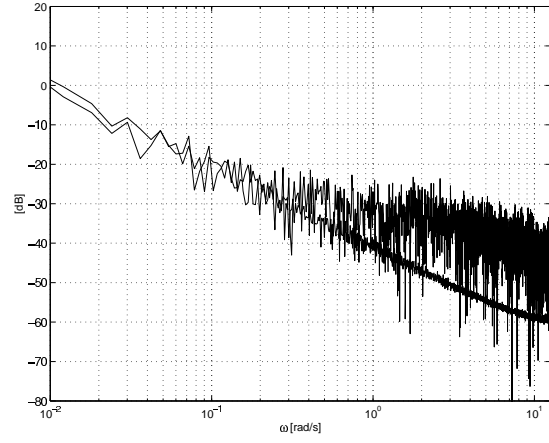


Fig. 3. Norms of the power spectral densities of  $\theta_o$  and  $\theta_g$ .

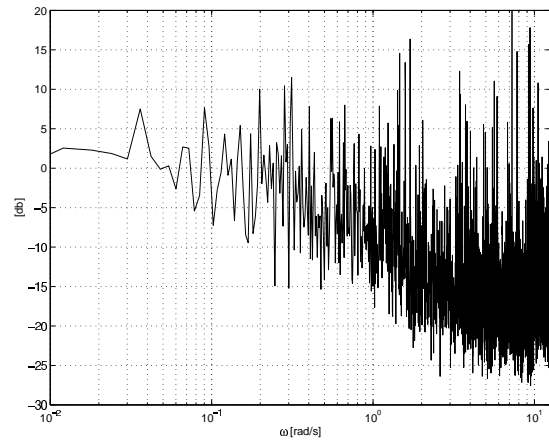


Fig. 4. Relation between the norms of the power spectral densities of  $\theta_o$  and  $\theta_g$ .

$$\theta_o(j\omega) = \frac{1}{j\omega}(\theta(j\omega) + a) \quad (12)$$

$$\theta_g(j\omega) = \frac{\theta(j\omega)}{j\omega} + b \quad (13)$$

where  $a$  and  $b$  are the RMS values of the velocity  $\varepsilon$  and position  $v$  noises respectively. Considering  $U$  to be an unity impulse and substituting equation (10) in equations (12) and (13), the relation between  $\theta_o$  and  $\theta_g$  is

$$\frac{\theta_o(j\omega)}{\theta_g(j\omega)} = \frac{k + a + aTj\omega}{k + bj\omega + bTj^2\omega^2} \quad (14)$$

The norms of the power spectral densities of  $\theta_o$  and  $\theta_g$  are represented in figure 3 and figure 4 shows the ratio between them. As can be seen in both, the best filter frequency is around 0.25 rad/s, for until then both sensors present almost the same measurements. It is for higher frequencies that the orientation from odometry and GPS significantly differ. A third-order low-pass filter with triple pole was chosen so that the noise was rapidly cutoff. Equation (15) now presents the correct form of  $G(s)$  shown in figure 2.

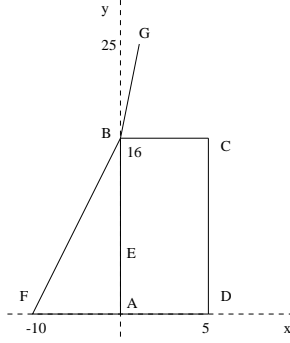


Fig. 5. Waypoints for test trajectories.

$$G(s) = \frac{1}{(4s + 1)^3} \quad (15)$$

#### 4. EXPERIMENTAL SETUP AND RESULTS

The above presented R-ANT robot was taken to an outdoor environment, on a parking place, mostly horizontal but with a ground surface presenting some irregularities which did not allow the robot to naturally proceed in straight lines and introduced some wandering around the planned line. The robot was equipped with a L1-L2 OEM3 10 channel Novatel GPS receiver operating for absolute position and velocity measurements, without any filtering. The robot was driven manually along different test trajectories in open loop, and the measurements from both odometry and the GPS receiver were logged on-board the mobile robot at a 4Hz sampling rate, for posterior comparison and fusion.

From the various trajectories used during the experiments, three illustrative examples are presented here, which show the respective advantages and drawbacks of the two navigation algorithms and of the compromise allowed by a complementary filtering fusion. In all the cases, the pretended trajectories were composed of straight segments between waypoints easily identified on the parking ground and according to figure 5.

##### 4.1 Rectangle path

The first selected example is a rectangular path, according to waypoints ABCDAE. The raw measurements are presented in figure 6, with the Cartesian coordinates and the orientation obtained from odometry and GPS. The first characteristic to be mentioned is the noisy signals given by the GPS receiver, with a random component around 1.7 m RMS on the position and 10 to 100 deg on the angle. This badly orientation estimate is due to the very low speed of the vehicle, which compared to the position error turns it difficult to estimate, despite a fair mean value, namely if it is compared with the odometry output.

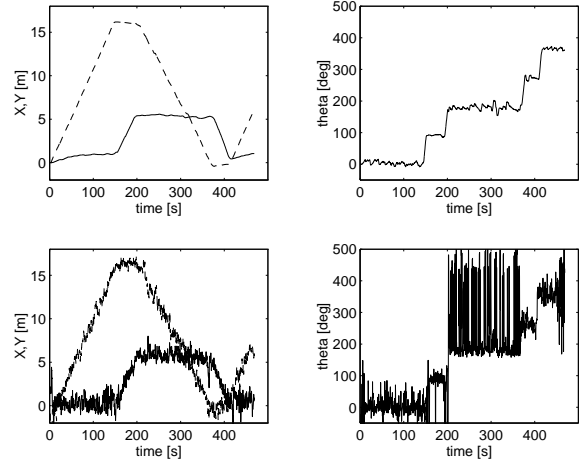


Fig. 6. Rectangle odometry (above) and GPS (below) measurements of position (left), with X in solid and Y in dashed, and orientation (right).

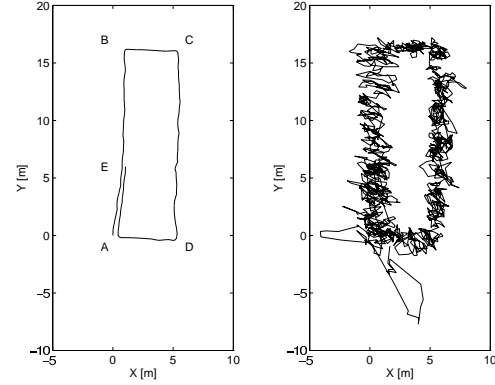


Fig. 7. Rectangle XY trajectory for odometry (left) and GPS (right).

The measured trajectories are presented in figure 7, showing here again the piecewise curve given by the odometry and noisy scattering of the GPS position. The data are however consistent one with the other, except for a small drift in the final of the odometry path, from A to E, with a small offset error in the x direction, and an GPS extra noise at the beginning.

The results obtained from the complementary filter output are presented in figure 8, with the equivalent curves. The trade-off between the raw measurements is clear, with some noise coming from the GPS data, but with the final drift corrected.

##### 4.2 Straight line path

The next example shows that also the complementary linear filter has its own drawbacks.

The path to be followed corresponds to the waypoints sequence ABGBABGBA. Raw data are shown in figure 9 and corresponding trajectories in figure 10. Here again it is possible to see the high-frequency noise in the GPS signal as well as the odometry drift. The output of the complementary filter in figure 11 shows

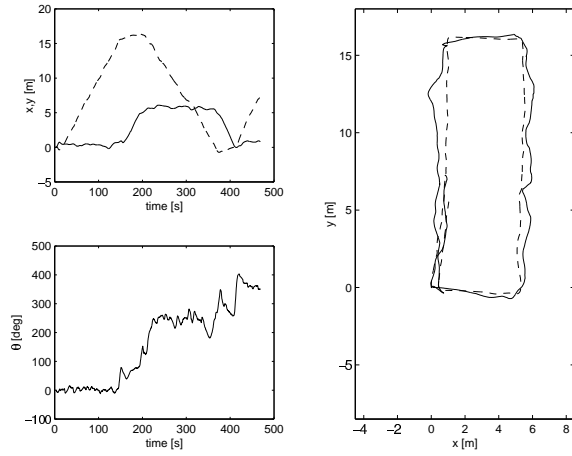


Fig. 8. Coordinates and orientation (left) and estimated path (right) output from complementary filter fusion.

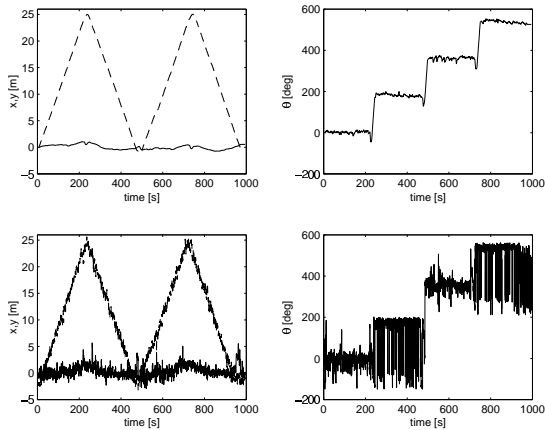


Fig. 9. Straight line odometry (above) and GPS (below) measurements of position (left) and orientation (right).

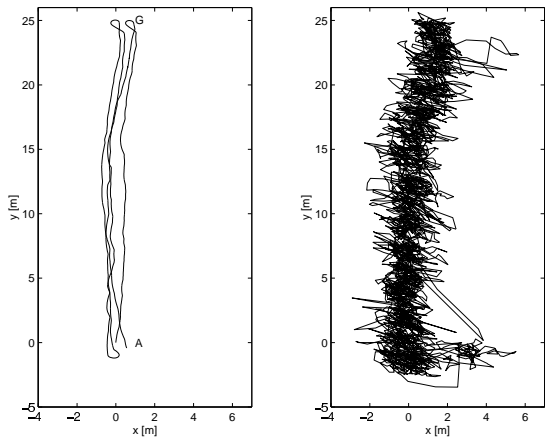


Fig. 10. Straight line XY trajectory for odometry (left) and GPS (right).

that when the GPS signal rambles away from the real trajectory, the complementary filter does not exclude these values for they are not in the high-frequency region, producing therefore an erroneous trajectory. Such cases must obviously be anticipated, taking into

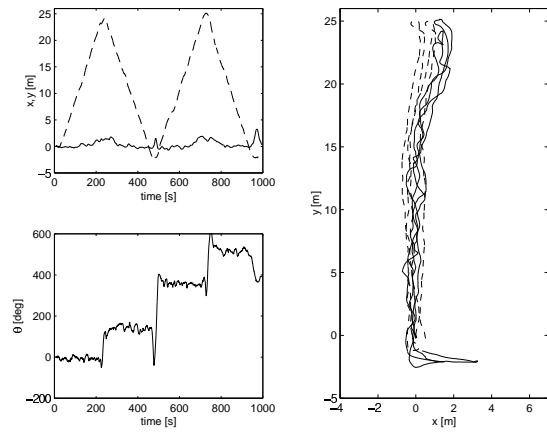


Fig. 11. Coordinates and orientation (left) and estimated path (right) output from complementary filter fusion.

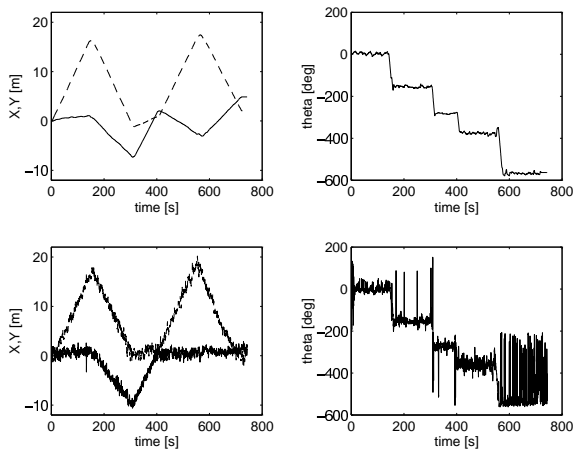


Fig. 12. Triangle odometry (above) and GPS (below) measurements of position (left) and orientation (right).

account the statistic and status data also available as GPS outputs.

#### 4.3 Triangular Path

The last example is probably more the example of a typical odometry drift (one can see in figure 13 that the trajectory given by odometry exhibits a heading error and is deviating from the FABAs actual trajectory), which a fusion system is expected to correct with the GPS absolute measurements. The path to be followed corresponds to the waypoints sequence ABFABA.

The resulting raw data are shown in figure 12. They are qualitatively very similar to the previous ones. But the trajectories estimated in figure 13 are here very different and clearly exhibit a drift in the odometry estimation, namely for the last part of the trajectory (ABA).

The output of the complementary filter, presented in figure 14, here again presents a compromise result, with some noise from GPS but with a fairly corrected drift of the ABA part of the trajectory (it must be

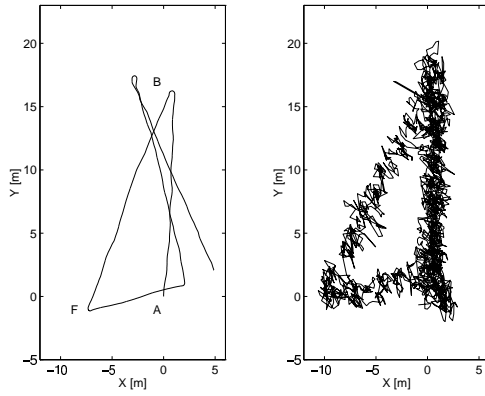


Fig. 13. Triangle XY trajectory for odometry (left) and GPS (right).

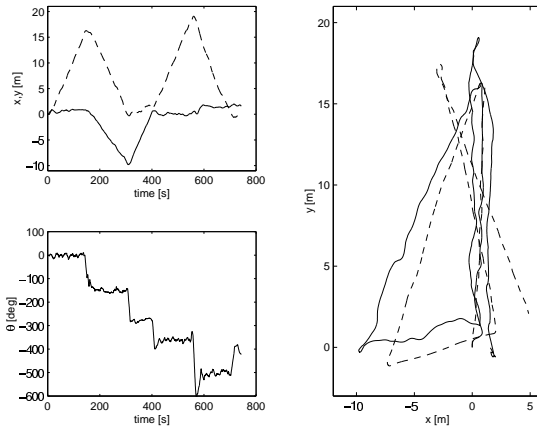


Fig. 14. Coordinates and orientation (left) and estimated path (right) output from complementary filter fusion.

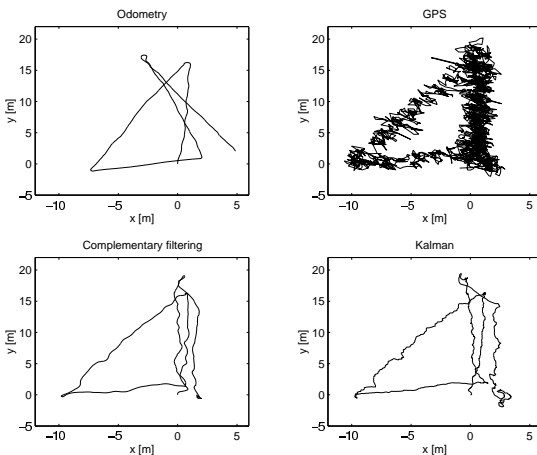


Fig. 15. Comparison results for the four systems: up left - odometry, up right - GPS, down left - complementary filter, down right - Kalman filter.

stressed that, as mentioned above, the robot piloting was not perfect and the segment following was indeed approximate).

In figure 15, comparison results including the ones obtained in (Moutinho and Azinheira 2003) by Kalman filtering, are shown.

## 5. CONCLUSION

The present paper introduced a feasible solution for the navigation of an outdoor autonomous mobile robot. Taking advantage of the complementary characteristics from the two sensor systems, the technique is based on the use of a complementary filter to fuse the data from the vehicle odometry and the raw position and velocity measurements output from a GPS receiver.

Experimental results are presented, which allow to compare the two original measurements, clearly showing the advantages and disadvantages of odometry or GPS alone, as well as of the filtered result. The output of the complementary filter fusion is in agreement with the searched compromise, canceling the odometry drift, to the expense of a little noise in the position estimate, as long as the GPS signal does not ramble. Again we emphasize that this method does not take into account the dynamic of the system.

After this comparison and validation study, next step is obviously to implement the fusion algorithm, close the loop and verify the correct path tracking of the mobile robot.

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