Actas do Encontro Científico
3º Festival Nacional de Robótica - ROBOTICA2003
Lisboa, 9 de Maio de 2003.

ROBOT FORMATIONS GENERATED BY NON-LINEAR ATTRACTOR DYNAMICS

Sergio Monteiro * **Estela Bicho** **

* *sergio.monteiro@dei.uminho.pt*
** *estela.bicho@dei.uminho.pt*

Dep. Industrial Electronics
University of Minho

Abstract: One underlying and fundamental issue in multi-robot systems is the control and coordination of several robots such that they keep a particular formation during movement. In this paper we focus on modelling formation of non-holonomic mobile robots using non-linear attractor dynamics. The benefit is that the behavior of each robot is generated by time series of asymptotically stable states which therefore contribute to the robustness against environmental perturbations. This study extends our previous work (Monteiro and Bicho, 2002). Here we develop a set of decentralized and distributed basic control architectures that allows each robot to maintain a desired position within a formation and to enable changes in the shape of the formation which are necessary to avoid obstacles. Simulation results for teams with four and six mobile robots are presented *.

* Project financed by the Portuguese Foundation for Science and Technology(POSI/SRI/38051/2001)

Keywords: mobile robots, formation control, attractor dynamics, attractors, repellers.

1. INTRODUCTION

In this paper we address the fundamental issues underlying the control and coordination of multiple autonomous mobile robots that must drive maintaining a desired geometrical formation and simultaneously avoid collisions with obstacles, in an unknown environment. This problem has received much attention from researchers working on cooperative robotics (see e.g Balch and Arkin (1998), Desai *et al.* (2001), Johnson and Bay (1995), Lewis and Tan (1997), Paulino and Araújo (2001), (Tabuada *et al.*, 2001), Yamaguchi (1999) and Wang (1995) for some works). The motivation is that there are many interesting applications that require the robots to coordinate and control their movements more closely (e.g. box pushing (Lewis and Tan, 1997), payload/object transportation (Johnson and Bay, 1995)(Soares and Bicho, 2002), capturing/enclosing an invader (Yamaguchi, 1999).

This paper extends our previous work (Monteiro and Bicho, 2002). There we have proposed a non-linear dynamical systems approach to behavior-based formation control. As a case study we have presented only the example of navigation in triangle formation for a team of three autonomous robots. In this previous study the distance was not controlled and velocity control, which is also important to maintain the configuration, was not explained formally. The flexibility and reconfigurability of our approach to formation control remained therefore an open question.

Here we develop a set of decentralized and distributed basic control architectures for line, column and oblique formation for a team of two robots. These dynamic control architectures can then be combined and generate more complex formations for larger teams of robots. In particular, we show teams of four and six mobile robots driving in line, column, diamond, star and hexagon. We demonstrate the flexibility of our dynamic control architectures by presenting the ability to avoid sensed obstacles integrated with movement in formation. Although we mainly present examples of formations with teams of four and six robots, more complex general configurations (larger number of robots) can be solved by our approach (Bicho and Monteiro, 2003, submitted)).

We assume that the robots have no prior knowledge of the environment and we follow a master-referenced strategy for each robot in the team (Balch and Arkin, 1998) (Desai *et al.*, 2001).

The control architecture of each robot is structured in terms of elementary behaviors. The individual behaviors and their integration are modelled by non-linear dynamical system and bifurcations are used to make design decisions around points at which a system must switch from one type of solution to another. The benefit is that the mathematical properties associated with the concepts (c.f. Section 2) enable system integration including stability of the overall behavior of the autonomous systems.

The movement of each robot in time is generated as a time series of attractor (i.e. asymptotically stable) states. The benefit is that asymptotical stability can be actively maintained and thus the systems are robust against perturbations.

The rest of the paper is structured as follows: In section 2 we show how control architectures for formation control (line, column and oblique) for teams of two robots can be modelled by attractor dynamics formulated at the level of heading direction. Next, in section 3 these are integrated with obstacle avoidance dynamics which is also defined at the level of heading direction. Section 4 presents the path velocity control. Simulation Results are presented in section 5 and show that more complex shapes of formations for larger teams of robots can be achieved. The paper ends in section 6 with conclusions and an outlook for future work.

2. ATTRACTOR DYNAMICS FOR ROBOT FORMATIONS

In this section we first present how basic and simple control architectures for teams of two robots that generate navigation in formation (e.g. line, column and oblique) can be built based on the so called *A dynamical systems approach to behavior-based robotics* (Large *et al.*, 1999) (Schöner and Dose, 1992) (Schöner *et al.*, 1995) (Bicho, 2000). With these basic control architectures, more complicated formations (e.g. square, polygon, star) can be achieved for larger teams of robots.

2.1 Two robots in line

Two robots are said to be in line formation if they drive side-by-side at a desired distance (see Figure 1).

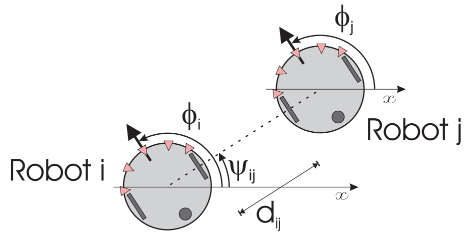

Fig. 1. Two robots in a line formation. $Robot_j$ is the *leader* of $Robot_i$ which must drive such that it sees its *leader* perpendicularly and simultaneously keep a desired distance, $d_{d,ij}$, between them.

A dynamical system for the heading direction of $Robot_i$ that generates line formation taking $Robot_j$ as a reference point is

$$\begin{aligned}\dot{\phi}_i &= f_{line,ij}(\phi_i) = \\ &= f_{app,ij}(\phi_i) + f_{div,ij}(\phi_i)\end{aligned} \quad (1)$$

where the terms $f_{app}(\phi_i)$ and $f_{div}(\phi_i)$ in the vector field define, respectively, attractive and divertive forces

$$f_{app,ij}(\phi_i) = -k_i k_{app,ij} \sin(\phi_i - (\psi_{d,ij} - \Delta\psi)) \quad (2)$$
$$f_{div,ij}(\phi_i) = -k_i k_{div,ij} \sin(\phi_i - (\psi_{d,ij} + \Delta\psi)) \quad (3)$$

where $\psi_{d,ij} = \psi_{ij} + \pi/2$.

The first contribution, $f_{app,ij}$ erects an attractor pointing $(\psi_{d,ij} - \Delta\psi)$ towards the $Robot_j$. The strength ($k_i k_{app,ij}$ with k_i fixed) of this attractor increases with the distance between the two robots, d_{ij}:

$$k_{app,ij}(d_{ij}) = \frac{1}{1 + e^{-\frac{d_{ij} - d_{desired,ij}}{\mu}}} \quad (4)$$

The second contribution, $f_{div,ij}$ erects an attractor $(\psi_{d,ij} + \Delta\psi)$ pointing away from $Robot_j$. The strength of this attractor increases if the distance between the two robots, d_{ij}, decreases:

$$k_{div,ij}(d_{ij}) = 1 - k_{app,ij}(d_{ij}) \quad (5)$$

This implies that from the superposition of these two attractive forces only one attractor state results. The value of the attractor is a continuous function of the distance between the two robots. When they are at the desired distance then the resultant attractor arises at the direction $\psi_{d,ij}$ (see Figure 2).

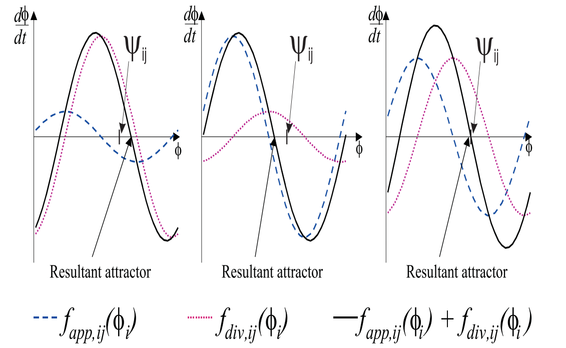

Fig. 2. This figure shows the two contributions to the line formation dynamics and their superposition for the three different physical situations. In the left plot $Robot_i$ is closer to $Robot_j$ than desired, then it must divert from $Robot_j$ (the divertive force is larger than the attractive force). The opposite situation is shown on the middle plot, with the distance being larger than desired. When the sensed distance equals the desired distance both $f_{app,ij}$ and $f_{div,ij}$ have the same value (right plot), causing $Robot_i$ to navigate in parallel to $Robot_j$. $\Delta\psi = \pi/4$ in Eq. 2 and 3.

2.2 Two robots in column

$Robot_i$ is said to drive in column formation with $Robot_j$ if it drives behind it at a desired distance (see Figure 3). To be in column formation, the

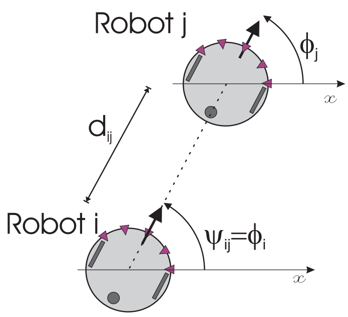

Fig. 3. Two robots in column formation.

follower must drive behind its *leader*, i.e. it must steer to the direction where it sees the *leader*. In terms of attractor dynamics this corresponds to place an attractor directly at the direction ψ_{ij} for the $Robot_i$'s heading direction dynamics:

$$\dot{\phi_i} = f_{col,ij}(\phi_i) = -k_{col}\sin(\phi_i - \psi_{ij}) \quad (6)$$

Where k_{col} defines the rate of relaxation of the heading direction to the attractor.

2.3 Two robots in oblique

We say that $Robot_i$ drives in oblique formation with respect to $Robot_j$ when during motion it maintains fixed (equal to a pre-defined angle θ_{ij}) the direction at which it sees $Robot_j$ (see Figure 4).

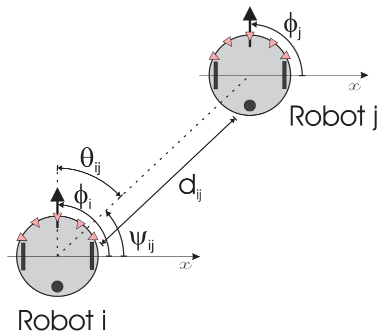

Fig. 4. Two robots in an oblique formation.

An oblique formation with respect to the *leader* can be reduced to a column formation with a *virtual leader robot* as illustrated in Figure 5.

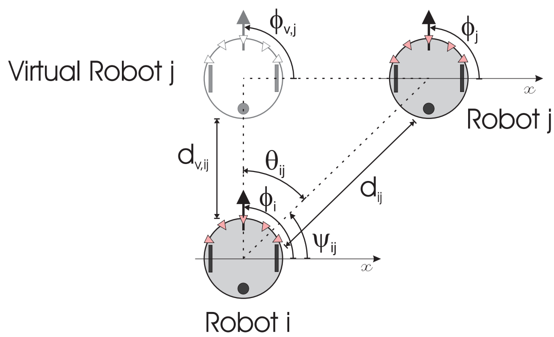

Fig. 5. Two robots in an oblique formation reduced to a column formation with a virtual leader robot.

If $d_{d,ij}$ is the desired distance to the real $Robot_j$ then the desired distance to the *virtual leader robot*, $d_{vd,ij}$, is

$$d_{vd,ij} = (d_{d,ij} + R_i + R_j)\cos(\theta_{ij}) - R_i - R_j \quad (7)$$

where R_i and R_j are the radius of $Robot_i$ and $Robot_j$, respectively. The heading direction dynamics for $Robot_i$ is then

$$\dot{\phi} = f_{oblique,ij}(\phi_i) = -k_{oblique}\sin(\phi_i - \psi_{v,ij}) \quad (8)$$

with $\psi_{v,ij}$ being the direction at which the *virtual leader* lies as *seen* from the current position of $Robot_i$:

$$\psi_{v,ij} = \arctan\left(\frac{d_{ij}\sin(\psi_{ij}) + d_{d,ij}\sin(\theta_{ij})\cos(\phi_{ij})}{d_{ij}\cos(\psi_{ij}) - d_{d,ij}\sin(\theta_{ij})\sin(\phi_{ij})}\right) \quad (9)$$

3. INTEGRATION WITH OBSTACLE AVOIDANCE

An obstacle avoidance dynamics formulated at the level of heading direction has been previously elaborated and implemented on a vehicle platform on which the simulated robots here are inspired (see Bicho2000:

$$\dot{\phi}_i = \sum_s f_{\mathrm{obs},s}(\phi_i) \tag{10}$$

where $f_{obs,s}$ are repulsive "force-lets, defined around each direction in which obstructions are sensed. These are characterized by (a) the direction, $\psi_{obs,s}$, to be avoided, (b) the strength, $\lambda_{obs,s}$, of repulsion, and (c) the range, σ_s over which repulsion acts. These repulsive force-lets can be straightforwardly erected by the distance sensors:

$$f_{obs,s}(\phi_i) = \lambda_{obs,s}(\phi_i - \psi_s)\exp[-\frac{(\phi_i - \psi_s)^2}{2\sigma_s^2}] \tag{11}$$

where $\psi_s = \zeta s + \phi$ is the direction in space into which an IR sensor, mounted at angle ζs from the frontal direction, is pointing. The strength of repulsion, $\lambda_{obs,s}$, is a decreasing function of sensed distance, d_s, to the obstruction, as estimated from the IR output with crude calibration. The functional form

$$\lambda_{obs,s} = \beta_1 \exp\left[-d_s/\beta_2\right] \tag{12}$$

depends on two parameters controlling overall strength (β_1) and spatial rate of decay (β_2).

The range

$$\sigma_s = \arctan\left[\tan(\frac{\Delta\zeta}{2}) + \frac{R_{\mathrm{robot}}}{R_{\mathrm{robot}} + d_s}\right] \tag{13}$$

is adjusted taking both sensor sector, $\Delta\zeta$, and the minimal passing distance of the robot (at size R_{robot} of the platform) into account.

Note that the right hand side of Eq. 11 really only depends on the distance measures, d_s, obtained from the sensors, not actually on ϕ_i (to see this, replace $\phi_i - \psi_s$ by θ_s, which is fixed).

Finally, because we have formulated all the behavioral dynamics at the level of heading direction the contributions that generate the basic formations and the contributions arising from the detected obstacles can be integrated, adding the corresponding contributions to the vector field. Additionally the heading direction dynamics is augmented by a stochastic force $equation) f_{stoch} = \sqrt{Q}\xi_n$equation) chosen as Gaussian white noise, ξ_n, of unit variance, so that Q is the effective variance of the force. This stochastic force is important for two reasons: to ensure escape from repppelers within a limited time and in addition models sensory and motor noise. The complete heading direction dynamics is:

$$\dot{\phi}_i = \sum_s f_{obs,s}(\phi_i) + \gamma_{line} f_{line,i}(\phi_i) + \tag{14}$$
$$\gamma_{col} f_{col,i}(\phi_i) + \gamma_{oblique} f_{oblique,i}(\phi_i) + f_{stoch}$$

where γ_{line}, γ_{col} and $\gamma_{oblique}$ are mutually exclusive boolean variables that determines which configuration is desired for the formation.

4. PATH VELOCITY CONTROL

In the previous sections we modelled the changes in the heading direction of a *follower* robot. Here we focus on the path velocity. There is a series of different possibilities to accomplish this, but in this paper we only present one of them. In any case, the follower's path velocity must be controlled so that this robot can maintain the desired formation (relative orientation and distance to its *leader*). Additionally, velocity control must be constrained by sensed obstructions. This can be accomplished by means of a dynamic system for the path velocity:

$$\dot{v}_i = \gamma_{obs} g_{obs}(v_i) + \gamma_{line} g_{line,i}(v_i) + \tag{15}$$
$$\gamma_{col} g_{col,i}(v_i) + \gamma_{oblique} g_{oblique,i}(v_i) + g_{stoch}$$

where each contributions sets an attractor at the desired path velocity, $v_{\mathrm{i,d}}$. g_{stoch} has the same functional form of 3 and also models motor and sensor noise.

When the robot's heading direction is inside the repulsion range created by sensed obstructions then the obstacle avoidance term dominate (i.e. $\gamma_{obs} = 1$, $\gamma_{line} = 0$, $\gamma_{col} = 0$ and $\gamma_{oblique} = 0$) and in this case the desired path value for the path velocity is:

$$v_{\mathrm{i,d}} = d_{\min}/T_{\mathrm{2c,obs}} \tag{16}$$

which tries to stabilize a particular time to contact, $T_{\mathrm{2c,obs}}$, with the obstacle. $d_{\min}$ is the minimum distance given by the distance sensors. Reversely, when no obstructions are sensed or the robot's heading direction is outside the repulsive effect of obstacle contributions then the particular desired value for the velocity depends on the desired configuration. For column and oblique formation the desired value for the path velocity is

$$v_{\mathrm{i,d}} = \begin{cases} v_{\mathrm{j}} - (d_{\mathrm{i,d}} - d_{\mathrm{i}})/T_{\mathrm{2c}} & \text{if } d_{\mathrm{i}} \geq d_{\mathrm{i,d}} \\ -v_{\mathrm{j}} - (d_{\mathrm{i,d}} - d_{\mathrm{i}})/T_{\mathrm{2c}} & \text{else} \end{cases} \tag{17}$$

Which makes the robot to accelerate or decelerated depending on the *leader*'s path velocity, v_{j}, and on the requirement to maintain the distance

$d_{\mathrm{i,d}}$ to the leader. The parameter T_{2c} permits also to control accelerations and decelerations such that the robot's movement is smooth. Finally for the line formation we can set the attractor for the velocity as

$$v_{\mathrm{i,d}} = v_{\mathrm{j}}. \tag{18}$$

5. SIMULATION RESULTS

The complete dynamic architectures were evaluated in computer simulations. These were generated by a software simulator written in MATLAB. We modelled the robotic platforms, based on the physical prototype in which the dynamic control architectures described in Bicho (2000) have been previously implemented. In simulation the robots are represented as triplets (x_i, y_i, ϕ_i), consisting of the corresponding two Cartesian coordinates and the heading direction. Cartesian coordinates are updated by a dead-reckoning rule $(\dot{x_i} = v_i cos(\phi_i),\ \dot{y_i} = v_i sin(\phi_i))$ while heading direction, ϕ_i, and path velocity, v_i, are obtained from the corresponding behavioral dynamics. All dynamical equations are integrated with a forward Euler method with fixed time step, and sensory information is computed once per each cycle. Distance sensors are simulated through an algorithm reminiscent of ray-tracing. The target information is defined by a goal position in space (i.e. $(x_{\mathrm{tar}}, y_{\mathrm{tar}})$). It is assumed here that all the *leader* robots broadcast their current velocity to the *followers*.

Several simulation runs, each with different formation configurations are presented. In Figures 6 and 7 are presented, respectively, snapshots of a line formation simulation and the corresponding plots of the heading direction dynamics for each robot in the team.

Figure 8 shows snapshots of a simulation where a switch between two different configurations occurs (square to column formation). A run with a diamond formation appears in Figure 9. A simulation with the robots navigating in column formation in a very cluttered environment is shown in Figure 10

In Figure 11 two different control architectures generate the same geometric configuration. Finally a simulation run with six robots is presented in Figure 12.

6. CONCLUSION AND FUTURE WORK

We have demonstrated how basic control architectures for line, column and oblique formation for a team of two robots can be modelled by non-linear dynamical systems. These can then be combined and generate more complex formation for larger teams of robots. In particular we have shown a team of four and six mobile robots driving in line, column, diamond, star and hexagon. One advantage of our planning system is that the computational complexity does not increase with the number of robots in the team nor with the number of obstacles in the environment. We have also demonstrated the flexibility of our dynamic control architectures by presenting the ability to avoid sensed obstacles integrated with movement in formation and explicit commanded changes in the shape of the formations.

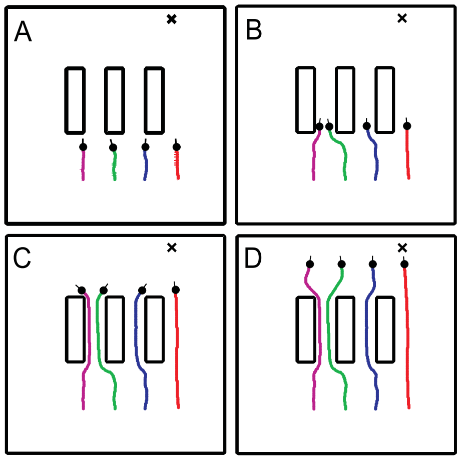

Fig. 6. Snapshots of a simulation of four robots in a line formation. $Robot_0$ navigates towards the target (marked by a cross), and leads $Robot_1$ (follower). $Robot_1$ leads $Robot_2$, which in its turn leads $Robot_3$. The robots depart almost in formation (A). Then they head towards the obstacle (B), and because the *obstacle avoidance* behavior takes precedence over the *formation* behavior, they step out of formation. As soon as they exit the corridors, and have no more obstacles, they restart the formation(C). They conclude in formation soon after (D).

The important contribution here, for formation control, is that the setpoints for the low level controller are generated by time series of attractor states.

For an example on how attractor dynamics can be used to design a distributed dynamic control architecture, that is also based on these elementary control architectures, and that enables a team of two robots to carry a long object and simultaneously avoid obstacles see Soares and Bicho (2002).

Due to the fact that we do not impose any constraints in the departure positions of the robots, neither we distinguish between static and moving obstacles [1], it might happen that the robots must

[1] e.g. a team mate might be an obstacle if sensed by distance sensors.

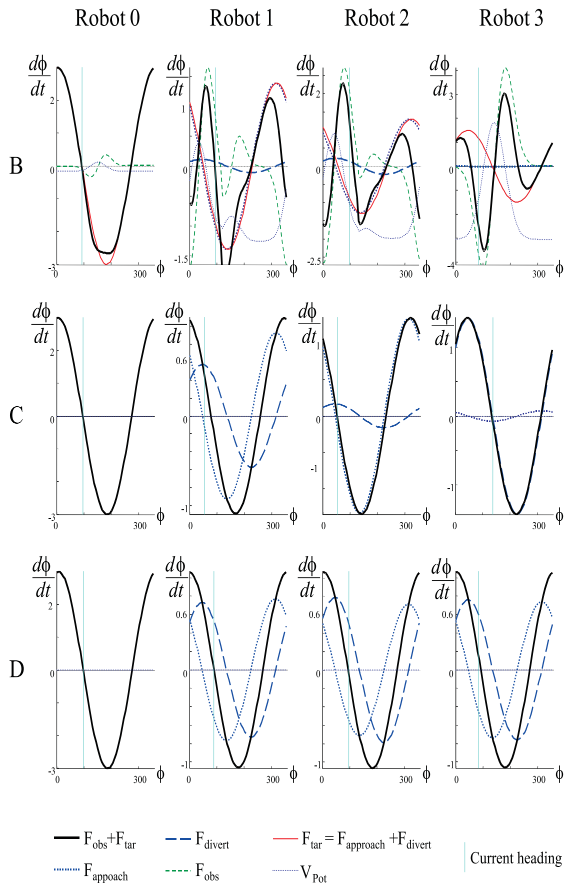

Fig. 7. Plots of the heading direction dynamics correspondent to panels B, C and D in Figure 6. Panel B: $Robot_0$ detects an obstacle to its left, but has low impact on the overall dynamics. The other robots also sense obstructions. Two attractors appear in the resultant dynamics, due to the superposition of the contributions of both behaviors *obstacle avoidance* and *keep formation*. Panel C: They are out of the corridor! No more obstructions are detected. They continue trying to set a formation. $Robot_2$ is more distant to its leader than it should, so f_{app} dominates over f_{div}. Conversely, $Robot_3$ should move away from $Robot_2$, because it is much closer than desired (the plot shows the dynamics equal to f_{div}). $Robot_1$ is closer to formation, as can be seen from the dynamics, where both terms are almost equal. Panel D: the robots reach formation! Note that in each plot the current value of the heading direction (indicated by the intersection of the vertical blue line with the axis of ϕ) is always very near to an attractor state of the resultant dynamics!

execute complicated manouvers and thus take long to achieve the desired geometric configuration. This can be solved by a hierarchical higher system that solves local conflict situations (e.g. by defining which robot stops to allow another to pass).

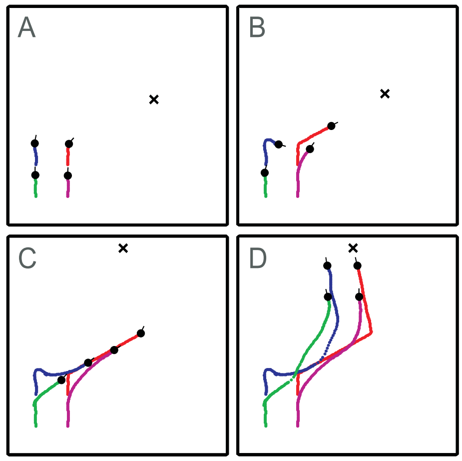

Fig. 8. Snapshots of a simulation where occurs transitions between formations. The robots start moving in a square formation. Then, an order to switch to a column formation is given (A) and they start to position themselves in the correct order (B) to reach the desired formation. The target changes location, and an order to change back to square is given (C). The robots are again in a square formation (D).

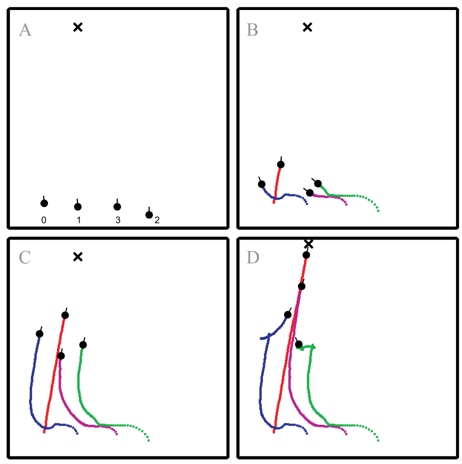

Fig. 9. Snapshots of a simulation of four robots acquiring a diamond formation, and then changing to column. They start placed in line.$Robot_0$ heads towards the target, marked by an X (A). An order to switch to diamond formation is given (B). In this formation, $Robot_1$ and $Robot_2$ should follow $Robot_0$ in an oblique formation. $Robot_1$ keeps to the left ($\theta_{10} = \pi/4$) and $Robot_2$ to the right ($\theta_{20} = -\pi/4$). $Robot_3$ follows $Robot_0$ in a column formation. When they reach diamond formation (C) an order to change to column is given. The robots try to position themselves such that the correct formation is achieved (D), i.e., $Robot_0$ leads the column, followed by $Robot_3$, which, in its turn, leads $Robot_1$ that is followed by $Robot_2$.

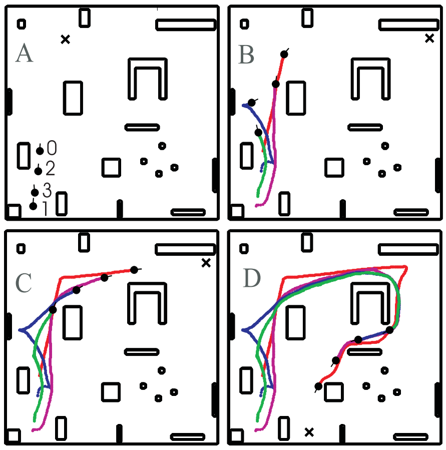

Fig. 10. Snapshots of a simulation run with the robots in column navigating among obstacles. The robots start in column, but in the wrong order (A). They try to regain the correct formation ($Robot_0$ leading $Robot1$, leading $Robot_2$, leading $Robot_3$)(B). The target location is moved to force the robots to navigate in the environment (C and D).

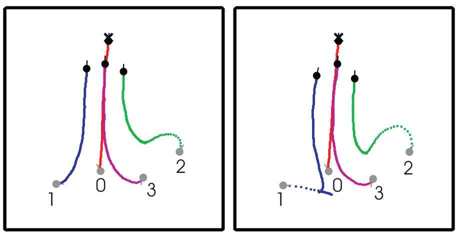

Fig. 11. Two diferent simulation runs with four robots acquiring the same star (geometrically). The team structure is different for the two simulation. For both simulations, $Robot_0$ is the team leader and $Robot_3$ follows it in a column formation. On the left simulation $Robot_1$ and $Robot_2$ follow $Robot_0$ in oblique (with $\theta_{10} = \pi/6$ and $\theta_{20} = -\pi/6$), while on the right simulation they follow $Robot_3$ (with $\theta_{13} = \pi/3$ and $\theta_{23} = -\pi/3$).

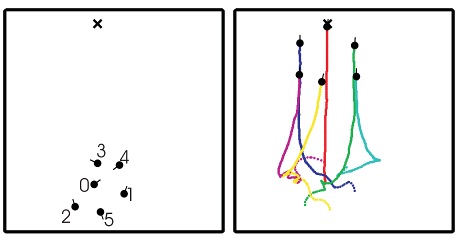

Fig. 12. Initial and final snapshots of a simulation run with six robots acquiring a hexagon formation. $Robot_0$ is the team leader.

In a near-future the complete architectures must be implemented and their performance evaluated in a team of physical mobile robots. Currently, we are initiating to implement these on a team of Khepera robots. Next, implementations on larger size robots will be also done. The implementation on different robot platforms will also permit to infer how easy it is to transfer our control architecture from one type of robots to another.

ACKNOWLEDGEMENTS

This project was supported, in part, through grants SFRH/BD/3257/2000 and POSI/SRI/38051/2001 to E.B. from the portuguese Foundation for Science and Technology (FCT). The contributions from Rui Soares are gratefully acknowledged. We are grateful to Carlos Oliveira e Ricardo Sa for their help with the simulator. We also would like to thank Isabel Ribeiro, from *Instituto de Sistemas Robóticos - IST* for the questions in Robotica'2001 concerning other geometric configurations.

REFERENCES

Balch, Tucker and R. C. Arkin (1998). Behavior-based formation control for multirobot teams. *IEEE Transactions on Robotics and Automation* **14**(6), 926–939.

Bicho, E (2000). *Dynamic Approach to Behavior-Based Robotics: design, specification, analysis, simulation and implementation.* Shaker Verlag. Aachen. ISBN 3-8265-7462-1.

Bicho, E and S Monteiro (2003, submitted). Formation control for multiple mobile robots: a non-linear attractor dynamics approach. In: *2003 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, submitted.*

Desai, J, J Ostrowski and V Kumar (2001). Modeling and control of formations of nonholonomic mobile robots. *IEEE Transactions on Robotics and Automation* **17**(6), 905–908.

Johnson, P and J Bay (1995). Distributed control of simulated autonomous mobile robot collectives in payload transportation. *Autonomous Robots* **2**(1), 43–64.

Large, E W, H I Christensen and R Bajcy (1999). Scaling the dynamic approach to path planning and control: Competition amoung behavioral constraints. *The International Journal of Robotics Research* **18**(1), 37–58.

Lewis, M A and K Tan (1997). High precision formation control of mobile robots using virtual structures. *Autonomous Robots* **4**, 387–403.

Monteiro, S and E Bicho (2002). A dynamical systems approach to behavior-based formation control. In: *Proc. IEEE Int. Conf. Robotics and Automation.* ICRA 2002. pp. 2606–2611.

Paulino, A and H Araújo (2001). Control aspects of maintaining non-holonomic robots in geometric formation. In: *Proceedings of the 9th International Symposium on Intelligent Robotic Systems.* SIRS2001. Toulouse, France. pp. 18–21.

Schöner, G and M Dose (1992). A dynamical systems approach to task-level system integration used to plan and control autonomous vehicle motion. *Robotics and Autonomous Systems* **10**, 253–267.

Schöner, G, M Dose and C Engels (1995). Dynamics of behavior: Theory and applications for autonomous robot architectures. *Robotics and Autonomous Systems* **16**, 213–245.

Soares, R and E Bicho (2002). Using attractor dynamics to generate decentralized motion control of two mobile robots transporting a long object in coordination. In: *Proc. of the workshop on Cooperative Robotics, in IROS 2002: 2002 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*. EPFL Lausanne, Switzerland.

Tabuada, P, G Pappas and P Lima (2001). Feasible formations of multi-agent systems. In: *Proc. of the American Control Conference*. Arlington, VA.

Wang, P K C (1995). Navigation strategies for multiple autonomous robots moving in formation. *Robotics and Autonomous Systems* **16**, 213–245.

Yamaguchi, H (1999). A cooperative hunting behavior by mobile-robot troops. *The International Journal of Robotics Research* **18**(8), 931–940.